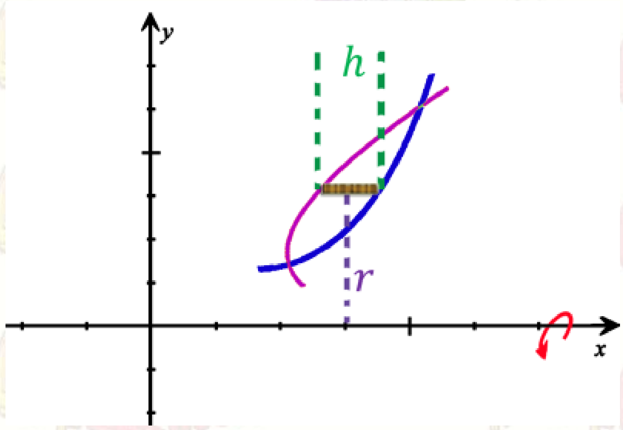
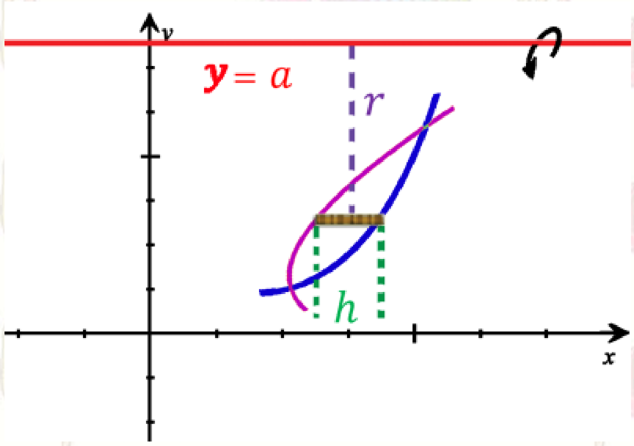
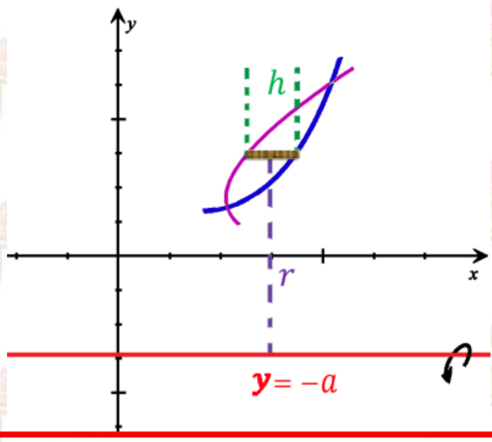
HOSSAM GHANEM

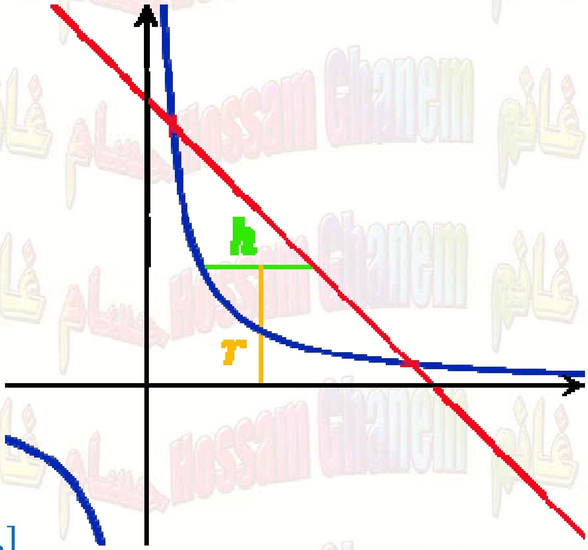
(50) 6.3 Volumes by Cylindrical Shells

	$x - axis$
Cylindrical Shells	$V = 2\pi \int_a^b rh\,dy$

Cylindrical Shells $x - axis$	$x_2 = f(y)$ $x_1 = g(y)$	
$V = 2\pi \int_a^b rh\,dy$	$h = x_2 - x_1$ $r = y$	
Cylindrical Shells $x - axis$	$x_2 = f(y)$ $x_1 = g(y)$	
$V = 2\pi \int_a^b rh\,dy$	$h = x_2 - x_1$ $r = a - y$	
Cylindrical Shells $x - axis$	$x_2 = f(y)$ $x_1 = g(y)$	
$V = 2\pi \int_a^b rh\,dy$	$h = x_2 - x_1$ $r = a + y$	

Example 1 14 January 6, 1996	the region in the first quadrant bounded by the graphs of the curves $y=\frac{2}{x}$ and $x+y=3$ is revolved abut the $x-axis$. Find the volume of resulting solid

Solution



$x=\frac{2}{y} \qquad x=3-y$

$h=x_2-x_1=3-y-\frac{2}{y}$

$r=y$

$$V=2\pi\int_1^2 rh\ dy=2\pi\int_1^2\left(3-y-\frac{2}{y}\right)y\ dy$$

$$=2\pi\int_1^2(3y-y^2-2)\ dy$$

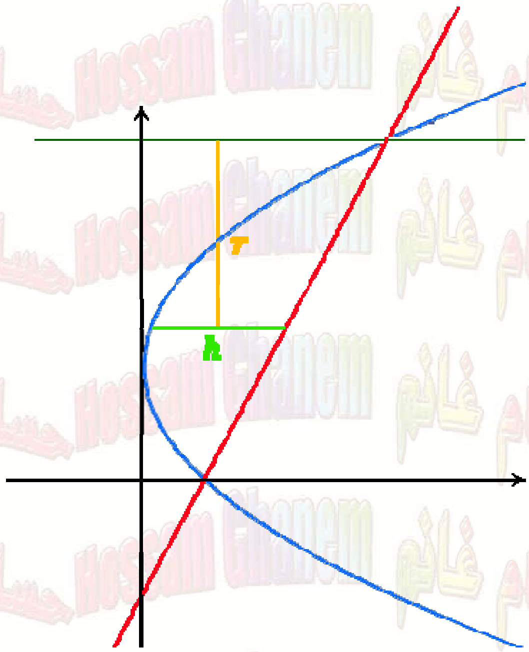
$$=2\pi\left[\frac{3}{2}y^2-\frac{1}{3}y^3-2y\right]_1^2$$

$$=2\pi\left[6-\frac{8}{3}-4-\left(\frac{3}{2}-\frac{1}{3}-2\right)\right]=2\pi\left[2-\frac{8}{3}-\frac{3}{2}+\frac{1}{3}+2\right]$$

$$=2\pi\cdot\frac{12-16-9+2+12}{6}=2\pi\cdot\frac{1}{6}=\frac{\pi}{3}$$

Example 2 25 January 12 .2003	Find the resulting volume when the region bounded by the graphs of the equation $x=(y-1)^2$, and $x=y+1$ about the line $y=3$

Solution



$y+1=(y-1)^2$

$y+1=y^2-2y+1$

$y^2-3y=0$

$y(y-3)=0$

$y=0 \quad , \quad y=3$

$h=x_2-x_1=y+1-(y-1)^2$

$\quad =y+1-y^2+2y-1=-y^2+3y$

$r=3-y$

$$V=2\pi\int_a^b rh\ dy=2\pi\int_0^3(3-y)(-y^2+3y)\ dy$$

$$=2\pi\int_0^3(-3y^2+9y+y^3-3y^2)\ dy$$

$$=2\pi\int_0^3(y^3-6y^2+9y)\ dy=2\pi\left[\frac{1}{4}y^4-2y^3+\frac{9}{2}y^2\right]_0^3$$

$$=2\pi\left(\frac{3^4}{4}-2(27)+\frac{9}{2}(9)\right)=2\pi\left(\frac{81-216+162}{4}\right)=\frac{27\pi}{2}$$

Example 3 29 June 4. 2007	The region bounded by the curves $x = y^2$ and $x = y^3$ is revolved abut the line $y = -3$. Set up an integral that can be used to find the volume of the resulting solid in each case

Solution

$y^3 = y^2$

$y^3 - y^2 = 0$

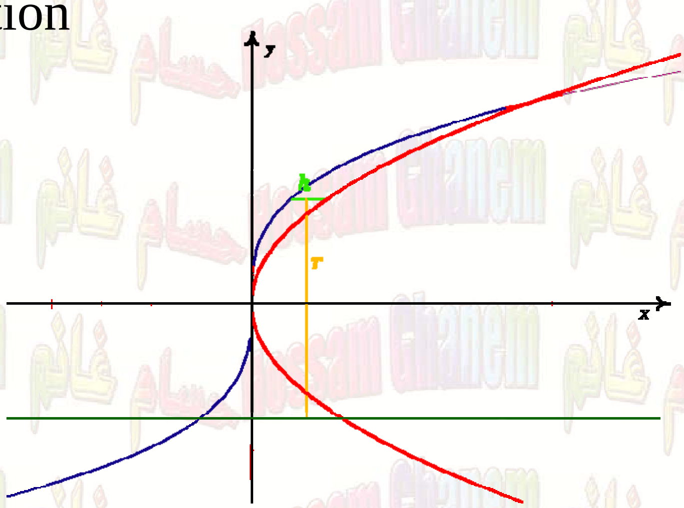
$y^2(y-1) = 0$

$y = 0 \quad , \quad y = 1$

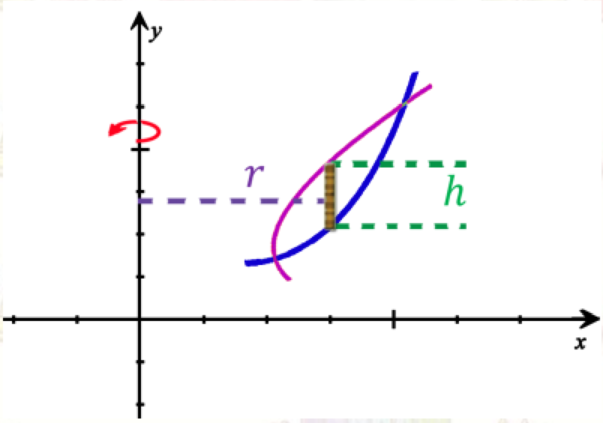
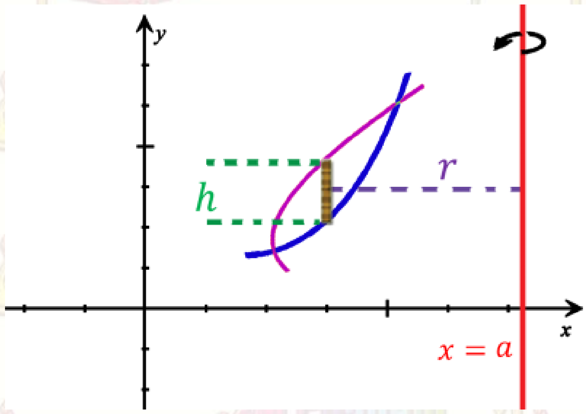
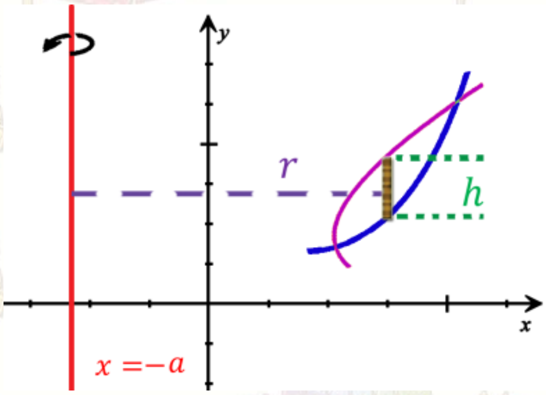
$h = x_2 - x_1 = y^2 - y^3$

$r = y + 3$

$$V = 2\pi \int_a^b rh \; dy = 2\pi \int_0^3 (y^2 - y^3)(y+3) \; dy$$



	$y-axis$
Cylindrical Shells	$V = 2\pi \int_c^d rh\,dx$

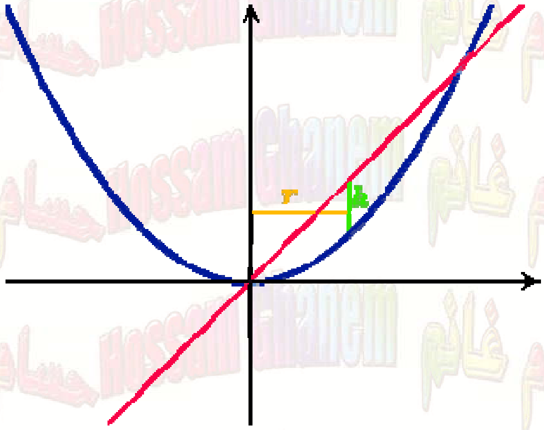
Cylindrical Shells $y-axis$	$y_2 = f(x)$ $y_1 = g(x)$	
$V = 2\pi \int_a^b rh\,dx$	$h = y_2 - y_1$ $r = x$	
Cylindrical Shells $y-axis$	$y_2 = f(x)$ $y_1 = g(x)$	
$V = 2\pi \int_a^b rh\,dx$	$h = y_2 - y_1$ $r = a - x$	
Cylindrical Shells $y-axis$	$y_2 = f(x)$ $y_1 = g(x)$	
$V = 2\pi \int_a^b rh\,dx$	$h = y_2 - y_1$ $r = a + x$	

Example 4	Set up an integral for the volume of the solid obtained when the region bounded by $y = x^2$ and $y = 4x$ is revolved abut $y - axis$.

Solution

$y = x^2 \quad , \quad y = 4x$
$x^2 = 4x$
$x^2 - 4x = 0$
$x(x-4) = 0$
$x = 0 \quad , \quad x = 4$
$h = y_2 - y_1 = 4x - x^2$
$r = x$

$$V = 2\pi \int_a^b rh \; dx = 2\pi \int_0^4 x(4x - x^2) \; dx$$

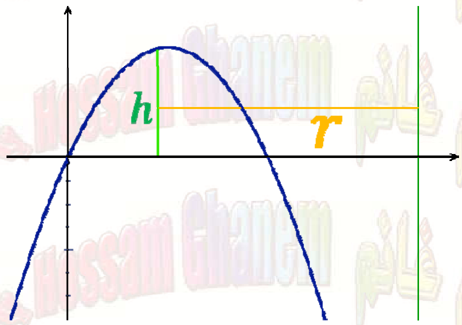


Example 5	21 May 27. 2001 Let R be the region bounded by the curves $y = 2x - x^2$, $y = 0$ Set up an integral that can be used to find the volume of the solid obtained by revolving R about the line $x = 5$

Solution

$2x - x^2 = 0$
$x(2-x) = 0$
$x = 0 \quad , \quad x = 2$
$h = y_2 - y_1 = 2x - x^2 - 0 = 2x - x^2$
$r = 5 - x$

$$V = 2\pi \int_a^b rh \; dx = 2\pi \int_0^2 (5-x)(2x - x^2) \; dx$$



Example 6	The region bounded by the graphs of $x^2 + y - 3 = 0$, $y - 3x + 1$, $x = 0$ and $x = 1$ is revolved abut the line $x = -4$. Find the volume of resulting solid

Solution

$y = 3 - x^2$, $y = 3x - 1$

$3x - 1 = 3 - x^2$

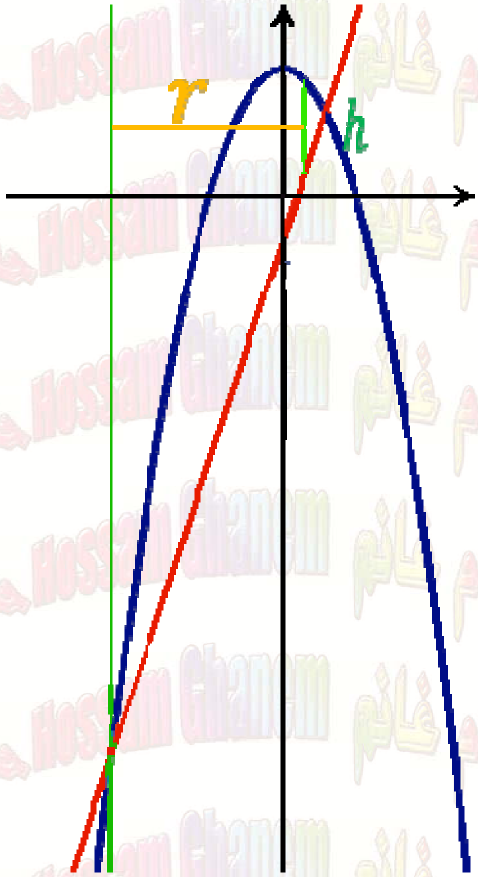
$x^2 + 3x - 4 = 0$

$(x + 4)(x - 1) = 0$

$x = -4$, $x = 1$

$h = (3 - x^2) - (3x - 1) = -x^2 - 3x + 4$

$r = x + 4$



$$V = 2\pi \int_a^b rh \; dx = 2\pi \int_{-4}^{1} (x + 4)(-x^2 - 3x + 4) \; dx$$

$$= 2\pi \int_{-4}^{1} (-x^3 - 3x^2 + 4x - 4x^2 - 12x + 16) \; dx$$

$$= 2\pi \int_{-4}^{1} (-x^3 - 7x^2 - 8x + 16) \; dx$$

$$= 2\pi \left[-\frac{1}{4} x^5 - \frac{7}{3} x^3 - 4x^2 + 16x \right]_{-4}^{1}$$

$$= 2\pi \left[-\frac{1}{4} - \frac{7}{3} - 4 + 16 - \left(-4^4 - \frac{7}{3} \cdot 64 - 64 - 64 \right) \right]$$